Quiz card – Possible strategies

Category	No.	Question	Answer (strategy)	Ref.
Breezy	1.	The three digits of a three-digit number add up to 25. How many such three-digit numbers are there?	6 799; 979; 997; 889; 898; 988	AMESA Mathematics Challenge 2006, Grade 7 Final Round
Breezy	2.	I multiplied two consecutive numbers (e.g. 4 and 5) on my calculator and got the answer 702. What is the sum of the two numbers?	53 $\sqrt{702} = 26{,}495\ldots$, therefore $26 \times 27 = 702$ and $26 + 27 = 53$	AMESA Mathematics Challenge 2004, Grade 7 Final Round
Breezy	3.	Split the number 18 into two whole numbers (e.g. 3 and 15). Now multiply these two numbers. What is the largest possible answer?	72	AMESA Mathematics Challenge 2004, Grade 7 Final Round
Breezy	4.	20 can be written as the sum of two squares, e.g. $20 = 4 + 16 = 2^2 + 4^2$ In how many different ways (ignore the order) can 85 be written as the sum of two squares?	2 ways $2^2 + 9^2 = 4 + 81$ $6^2 + 7^2 = 36 + 49$	AMESA Mathematics Challenge 2002, Grade 7 Final Round
Breezy	5.	How many two-digit numbers are there with both digits even?	20 20, 22, 24, 26, 28, (5) 40, 42, 44, 46, 48, (5) 60, 62, 64, 66, 68, (5) 80, 82, 84, 86, 88 (5)	AMESA Mathematics

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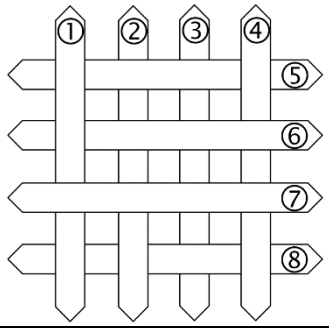
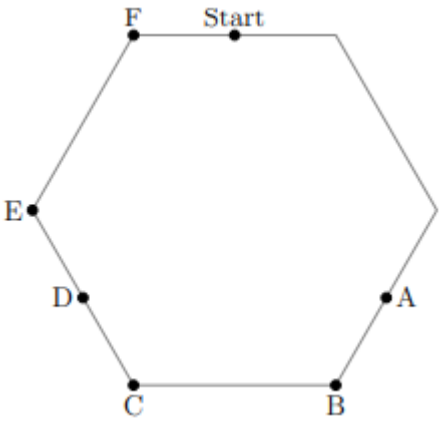
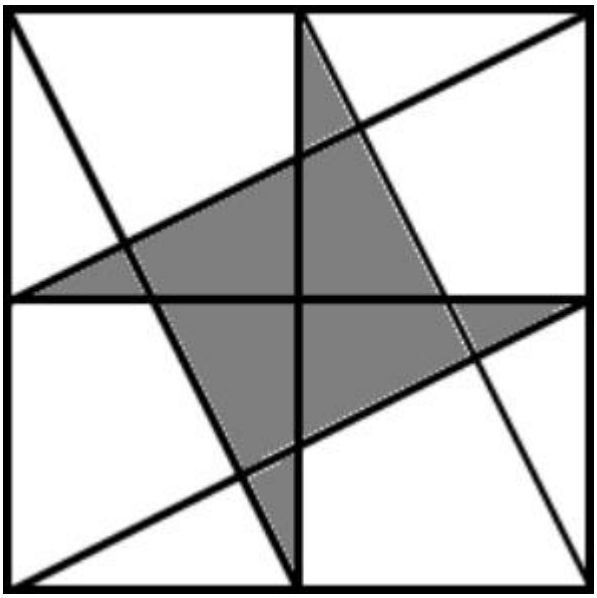
Breezy	6.	The sketch shows eight lolly sticks. If you must pick up the top one each time, in what order will you pick them up? 	7; 1; 6; 4; 5; 2; 8; 3	AMESA Mathematics Challenge 2001, Grade 7 Final Round
Breezy	7.	What is the smallest ten-digit number that has exactly two digits that are the same and all other digits are different? Note: Numbers cannot start with zero!	1 002 345 678	Wits Mathematics Competition 2022 Grade 6/7 Final Round
Breezy	8.	The digits of the year 2020 add up to 4. In how many other years (starting from 1), before 2020, has this happened?	27 They are 4, 13, 22, 31, 40, 103, 112, 121, 130, 202, 211, 220, 301, 310, 400, 1003, 1012, 1021, 1030, 1102, 1111, 1120, 1201, 1210, 1300, 2002 and 2011. To avoid missing any it's helpful to break into cases and sub-cases. For example those numbers which are less than 4 digits (broken into the subcases of what is in the hundreds column), which numbers are four digits long and begin with a 1 and which are four digits long and begin with a 2	WITS Mathematics Competition 2020, Gr 6/7 Qualifying Round

Category	No.	Question	Answer (strategy)	Ref.
Breezy	9.	There are 60 learners in a class. Always two students share a desk. Every boy shares a desk with a girl. Exactly half the girls share a desk with a boy. How many boys are in the class?	20 There are as many boys as 'half the girls' (because they share desks). Boys are therefore a third of the class.	Wits Mathematics Competition 2022 Grade 5/6 Final Round
Breezy	10.	Terri's birthday is on 3 December. On 1 August 2012 she was 11-years-old. In which year will she have her 21st birthday?	2021 She turned 12 in 2012	Wits Mathematics Competition 2021 Grade 5/6 Qualifying
Breezy	11.	A running track is in the shape of a hexagon with equal sides. Kerri started running from the start flag. She ran an anti-clockwise direction (that is towards F) and stopped one-third of the way around the track. At which point (A, B, C, D or E) did she stop?  	D Break the hexagon into 12 pieces, consisting of each half of each side. Count out 4 of these.	Wits Mathematics Competition 2021 Grade 5/6 Qualifying

Category	No.	Question	Answer (strategy)	Ref.
Breezy	12.	The inter-school soccer league consists of 8 teams. How many matches will each team play if every team plays against every other team twice – once at home and once away?	14 Each team will play 7 other teams (a team cannot play against itself), so each team plays 7 matches. However, this is done twice (home and away).	AMESA Mathematics Challenge 2014, Grade 7 Final Round
Breezy	13.	Mr Safe has a 4-digit combination that opens his lock. He remembers that the four digits are 3, 5, 7 and 9, but he has forgotten the correct order. What is the most different combinations that he must try to open the safe?	24 Make a systematic list, e.g. 3579; 3597, 3759; 3795, 3957; 3975, 9375; 9357, 9537 … Or: He has 4 choices for the first number, then 3 choices for the second, 2 for the third and 1 for the fourth. So, $4 \times 3 \times 2 \times 1$	AMESA Mathematics Challenge 2014, Grade 7 Final Round

Category	No.	Question	Answer (strategy)	Ref.
So-So	14.	In the diagram, a corner of the shaded star touches the middle of each side of the large square.  What fraction of the large square is NOT shaded?	$\frac{3}{4}$ The diagram can be split into four identical (up to rotation) squares. Each of these has $\frac{1}{4}$ shaded. So $\frac{3}{4}$ of each smaller square is unshaded and so $\frac{3}{4}$ of the original square is unshaded.	WITS Mathematics Competition 2020, Gr 6/7 Qualifying Round
So-So	15.	Today is a Monday. Thabo starts to read a book with 290 pages today. On Mondays he reads 25 pages and on every other day he reads 4 pages. On which day of the week does he finish reading the book?	Saturday Observe that Thabo reads a total of 49 pages a week. After six weeks (ending in a Sunday) he'd have read 296 pages if the book was long enough. The Saturday before he'd be on 292 pages and the Friday before on 288. Therefore he finishes on the sixth Saturday.	Wits Mathematics Competition 2022 Grade 6/7 Final Round

Category	No.	Question	Answer (strategy)	Ref.
So-So	16.	There are 14 people at a party. Every pair of people shakes hands exactly once. How many handshakes occur?	91 For each handshake there are 14 choices for the first person and 13 choices for the second person. So there are 14×13 choices. However, since we can choose the same pair of people in two different ways, the number of handshakes is $\frac{14 \times 13}{2} = 91$	Wits Mathematics Competition 2019 Grade 4/5 Final Round
So-So	17.	A painter takes two days to paint a room (all four walls and the ceiling). If he works at the same pace, how many days will he take to paint a room that is twice as wide, twice as long and twice as high?	8 days Take for e.g. a $2 \times 2 \times 2$ room. To paint each of the wall he will paint an area of 2×2 as well as another 2×2 area for the roof, therefore $5 \times (2 \times 2) = 20$. If you double all the measurements it will be 5(4 × 4) = 80. 80 is four times more than 20. So it will take him four times longer to paint the bigger room.	AMESA Mathematics Challenge 2001, Grade 7 Final Round
So-So	18.	Check this number pattern: $1 = 1 \times 1$ $1 + 3 = 2 \times 2$ $1 + 3 + 5 = 3 \times 3$ $1 + 3 + 5 + 7 = 4 \times 4$ Now calculate 1 + 3 + 5 + 7 + … all the way up to … + 97 + 99	2 500 50×50	AMESA Mathematics Challenge 2007, Grade 7 Final Round

Category	No.	Question	Answer (strategy)	Ref.
So-So	19.	What is the difference between the sum of the even numbers and the sum of the odd numbers from 1 to 100, both included?	50 $(2+4+6+8+\cdots+98+100)-(1+3+5+\cdots+97+99)$ $=(2-1)+(4-3)+(6-5)+\cdots+(98-97)+(100-99)$ $=1+1+1\ldots 50$ times	AMESA Mathematics Challenge 2008, Grade 7 Final Round
So-So	20.	What is the 83rd number in the following pattern: 1 ; 3 ; 5 ; 7 ; ....?	165 n^{th} number $=2\times n-1$, so 83^{rd} number $=2\times 83-1$	AMESA Mathematics Challenge 2011, Grade 7 Final
So-So	21.	Consider the following pattern: $1^3+2^3=3^2$ $1^3+2^3+3^3=6^2$ $1^3+2^3+3^3+4^3=10^2$ $1^3+2^3+3^3+4^3+\ldots+10^3=n^2$ What is the value of n?	55 Note that 1 + 2 = 3; 1 + 2 + 3 = 6; 1 + 2 + 3 + 4 = 10 So n = 1 + 2 + 3 + 4 + 5 + ... + 10 = 55	AMESA Mathematics Challenge 2014, Grade 7 Final Round

<table>
<tr><th>Category</th><th>No.</th><th>Question</th><th>Answer (strategy)</th><th>Ref.</th></tr>
<tr><td>So-So</td><td>22.</td><td>The Olympic committee printed numbered bibs for each of the athletes competing in the ski jump event, starting from the number 1. If 234 digits were printed overall, how many athletes competed?</td><td>114

The first 9 athletes use a single digit per bib. The next 90 use two digits a bib. Which means after printing 99 bibs we'll have use 189 digits and have 45 'left'. As the next 900 will use three digits a bib we can see that our remaining digits are enough for another 15 bibs. A total of 99 + 15 = 114.</td><td>Wits Mathematics Competition 2022 Grade 6/7 Qualifying Round</td></tr>
<tr><td>So-So</td><td>23.</td><td>The product of two positive integers is equal to twice their sum. The same product is also equal to six times the difference between the two integers. What is the sum of the integers?</td><td>9

As the numbers are small this can be solved by trial and error. A more systematic approach is to use algebra. Call the numbers a and b and choose $a < b$. Then $ab = 6b - 6a = 2a + 2b$. $6b - 6a = 2a + 2b$ implies $b = 2a$. So subbing into $ab = 6b - 6a$ gives $2a^2 = 6a$ and $a^2 = 3a$. Which gives us $a = 3$ and $b = 6$.</td><td>Wits Mathematics Competition 2022 Grade 6/7 Qualifying Round</td></tr>
<tr><td>So-So</td><td>24.</td><td>What is the value of a in the table?
<table><tr><td>1</td><td>2</td><td>3</td><td>4</td><td>…</td><td>a</td></tr><tr><td>4</td><td>6</td><td>8</td><td>10</td><td>…</td><td>64</td></tr></table></td><td>31</td><td>AMESA Mathematics Challenge 2012, Grade 7 Final Round</td></tr>
</table>

Category	No.	Question	Answer (strategy)	Ref.
So-So	25.	Which one of the following numbers will appear in the sequence: 2; 5; 10; 17; 26; 37; ...? (A) 901 (B) 902 (C) 903 (D) 904 (E) None of these	(A)901 Look for structure and pattern! N1 = 2 = 1×1 + 1 N2 = 5 = 2×2 + 1 N3 = 10 = 3×3 + 1 N4 = 17 = 4×4+ 1 Test the numbers! 30×30 + 1 = 901 is the only one which fits the pattern	AMESA Mathematics Challenge 2012, Grade 7 Final Round
So-So	26.	A series of 10 books were published at 2-year intervals. The sum of the publication years is 20 000. When was the first book published?	1991 The average year of publication was 2000. This would be the time midway between the publication of the fifth and sixth books. So the books were published in: 1991, 1993, 1995, 1997, 1999, 2001, 2003, 2005, 2007, and 2009.	Wits Mathematics Competition 2022 Grade 6/7 Final Round
So-So	27.	How many of the 5-digit numbers which consist of the five digits 1, 2, 3, 4 and 5 are divisible by all of 1, 2, 3, 4 and 5?	None To be divisible by 5, the last digit must be 5. But to be divisible by 2, the last digit must be 2 or 4. So none of these numbers can be divisible by 2 and 5, so none of them can be divisible by 1, 2, 3, 4, and 5.	AMESA Mathematics Challenge 2013, Grade 7 Final Round

Category	No.	Question	Answer (strategy)	Ref.
So-So	28.	Arnie, Bender and Cross are three robots. They are weighed two at a time. Here are the results: • A + B = 12 kg • B + C = 14 kg • C + A = 16 kg How much will all three weigh together?	21 kg Add all together: 2A + 2B + 2C = 42, so A + B + C = 21	AMESA Mathematics Challenge 2013, Grade 7 Final Round
Fiendish	29.	Calculate: $\frac{1 + 3 + 5 + 7 + \ldots + 97 + 99}{2 + 4 + 6 + 8 + \ldots + 98 + 100}$	$\frac{50}{51}$ Take special cases, be systematic, and notice the patterns: 1 number: $\frac{1}{2} = \frac{1}{2}$ 2 numbers: $\frac{1+3}{2+4} = \frac{4}{6} = \frac{2}{3}$ 3 numbers: $\frac{1+3+5}{2+4+6} = \frac{9}{12} = \frac{3}{4}$ Etc. Alternatively, if you know or develop some formulas: $\frac{50^2}{50 \times 51}$	AMESA Mathematics Challenge 2013, Grade 7 Final Round
Fiendish	30.	Xolile has a bag of marbles. He gave $\frac{1}{3}$ of them to Baba and then $\frac{1}{4}$ of the remaining marbles to Sam. If there are now 24 marbles in the bag, how many marbles did Xolile give to Baba?	16 marbles Suppose Xolile had x marbles. After giving $\frac{1}{3}$ to Baba, he had $\frac{2}{3}$ remaining; or $\frac{2}{3}$ of x. After giving $\frac{1}{4}$ of the remainder to Sam, he had $\frac{3}{4}$ of them left, or $\frac{3}{4}$ of $\frac{2}{3}$ of x which equals 24. So, $\frac{1}{2}$ of x equals 24, so $x = 48$. This means she gave Baba $\frac{1}{3}$ of 48	AMESA Mathematics Challenge 2013, Grade 7 Final Round

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<tr><td>Fiendish</td><td>31.</td><td>Numbers are arranged in the following patterns:

1 2 3 4 5 6 row 1
7 8 9 10 11 12 row 2
13 14 15 16 17 18 row 3
… … … … … … row 4

What will the third number in row 81 be?</td><td>403

3rd number in nth row $= 5 \times n - 2$, so 3rd number in 81st row $= 5 \times 81 - 2$</td><td>AMESA Mathematics Challenge 2005, Grade 7 Final Round</td></tr>
<tr><td>Fiendish</td><td>32.</td><td>All the counting numbers are arranged in columns as shown below.

<table><tr><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th><th>F</th><th>G</th></tr><tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td></tr><tr><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td><td>21</td></tr><tr><td>…</td><td>…</td><td>…</td><td>…</td><td>…</td><td>…</td><td>…</td></tr></table>
In which column is 500?</td><td>C

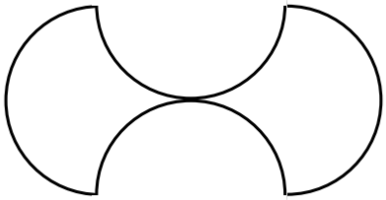
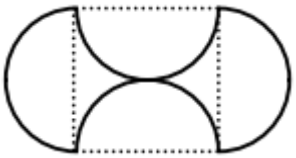
Note that column G consists of multiples of 7. If we divide 500 by 7 we get 71 remainder 3. So 497 will be in column G, 498 will be in column A, 499 in column B and 500 in column C</td><td>AMESA Mathematics Challenge 2014, Grade 7 Final Round</td></tr>
<tr><td>Fiendish</td><td>33.</td><td>a, b, c and d are four adjacent dates in a calendar as shown.

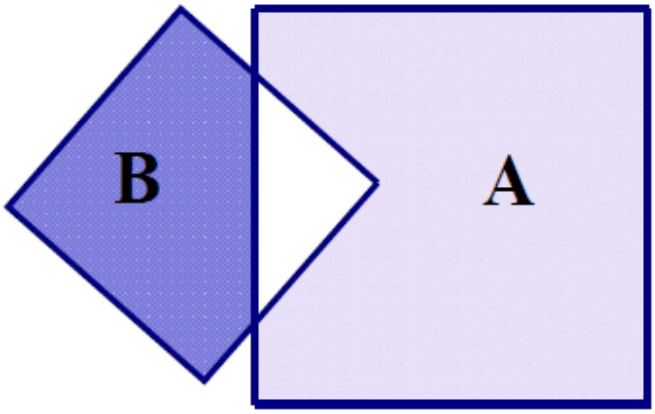
<table><tr><th>Mon</th><th>Tue</th><th>Wed</th><th>Thu</th><th>Fri</th><th>Sat</th><th>Sun</th></tr><tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td>a</td><td>b</td><td></td><td></td><td></td></tr><tr><td></td><td></td><td>c</td><td>d</td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr></table>
Which statement is true for any calendar?

(A) $a + c = b + d$ (B) $a + d = b + c$
(C) $c - b = a - d$ (D) $a + b = c + d$
(E) $d - a = c + b$</td><td>B

We know that $c - a = d - b = 7$ so $c - a = d - b$. So $a + d = b + c$</td><td>AMESA Mathematics Challenge 2014, Grade 7 Final Round</td></tr>
</table>

Category	No.	Question	Answer (strategy)	Ref.
Fiendish	34.	a, b, c and d are four adjacent dates in a calendar as shown. Mon | Tue | Wed | Thu | Fri | Sat | Sun — | — | — | — | — | — | — | | | | | | | | | | a | b | | | | | | c | d | | | | | | | | | | What is the value of a if $a + b + c + d = 52$?	9 $a + b + c + d = 52$ $a + (a + 1) + (a + 7) + (a + 8) = 52$ $4a = 36$ $a = 9$	Aarnout's version
Fiendish	35.	30 students wrote a mathematics exam out of 100 marks. The average score was 50. The average score of those who passed was 60 and the average score of those who failed was 45. How many students passed?	10 The total number of marks scored was 30×50 = 1500. If x students passed then $1500 = 60x + 45(30 - x)$ which solves to $x = 10$. This could also be solved via trial and error if the student was not familiar with algebra (trying different values for the number of passers).	Wits Mathematics Competition 2022 Grade 6/7 Qualifying Round
Fiendish	36.	The average of eleven numbers is 8. If a twelfth number is added to these numbers, the average of all twelve numbers is now 11. What is the twelfth number?	44 We know: $\frac{\text{Sum of numbers}}{11} = 8$, so Sum of numbers = 11 × 8 = 88 If the new number is x, then $\frac{88+x}{12} = 11$. So $x = 12 \times 11 - 88$	AMESA Mathematics Challenge 2013, Grade 7 Final
Fiendish	37.	A bath fills in 12 minutes if the plug is in. It empties in 18 minutes when the tap is off. If the tap is running and the plug is out, how long will it take to fill the bath?	36 minutes Filling: In 1 minute $\frac{1}{12}$ of bath fills. Emptying: In 1 minute $\frac{1}{18}$ of bath empties Together: In 1 minute $\frac{1}{12} - \frac{1}{18} = \frac{1}{36}$ of bath fills. So the whole bath $\left(\frac{36}{36}\right)$ fills in <u>36 minutes</u>	AMESA Mathematics Challenge 2006, Grade 7 Final Round

Category	No.	Question	Answer (strategy)	Ref.
Fiendish	38.	The figure is a combination of four semi-circles, each with a radius of 3 cm. What is the area of the figure? 	36 cm^2 The area of the semi-circles on the left and right side is equal to the area of a full circle. $\pi r^2 = \pi 3^2 =$ 28,27433… cm^2 …c The area of the square (see the sketch below) is 6 × 6 = 36 cm^2. But we need the area of the square minus the area of the 2 semi-circles: 36 cm^2 – 28,27433 cm^2 = 7,725666… cm^2 … 28,27433 + 7,725666 = 36 cm^2 	AMESA Mathematics Challenge 2001, Grade 7 Final Round
Fiendish	39.	A water tank is $\frac{7}{8}$ full. After 420 litres had been drawn from it, it is half full. How many litres does the tank hold when it is full?	1 120 litres $\frac{7}{8} - \frac{1}{2} = \frac{3}{8} = 420$ litres, so $\frac{1}{8} = 240$ litres $\div 3 = 140$ litres. So the full tank $= \frac{8}{8} =$ 140 litres × 8 = 1120 litres	AMESA Mathematics Challenge 2013, Grade 7

Category	No.	Question	Answer (strategy)	Ref.
Fiendish	40.	Two squares with lengths 4 cm and 6 cm respectively, partially overlap as shown in the diagram. What is the difference between shaded area A and shaded area B?  	20 cm^2 Let the overlapping region have an area of x cm^2. Area of larger square is 36 cm^2, So area $A = 36 - x$ Area of smaller square is 16 cm^2, so area $B = 16 - x$ area A − area B = $(36 - x) - (16 - x) = 36 - x - 16 + x = 36 - 16 = 20$	AMESA Mathematics Challenge 2013, Grade 7 Final Round