DIFFERENTIATION

FIRST DRAFT

TEACHER GUIDE

CHAPTER 8.2

Brombacher & Associates

EXTENSION ACTIVITIES

The need for differentiation in a NumberSense Mathematics lesson

Learning opportunities not presented within a child's zone of proximal development (Vygotsky, 1978) cannot be effective. Piaget (1953) stated this more forcefully as "giving no education is better than giving it at the wrong time."

In the NumberSense Mathematics Programme, we recognise that mathematics classes are not homogeneous with respect to the mathematical proficiency levels of the children.

We reject outright the characterisation of children as weak and strong. We prefer to think of children as being on a continuum. On the one end of the continuum, are those children who have not yet achieved the level of mathematical proficiency we expect of children at their age. On the other end, children whose mathematical proficiency already exceeds the level we expect of children at their age. We hold the position that those who have not yet achieved the level of mathematical proficiency we expect of children at their age, can reduce the gap between where they are and where we expect them to be.

We recognise that there are Children with Special Needs (CWSN). CWSN are, in general, those who have some type of disability and require exceptional care and extra help. In the context of learning, CWSN are children who require additional support and accommodations to succeed in their education due to a range of possible challenges. This discussion does not address the support that CWSNs require.

The characterisation of children as weak and/or strong results in teachers treating, in the way they teach them, the so-called weak children as if they cannot achieve the mathematical proficiency we expect of them. This, in turn, leads to the manifestation of the expectation they have of those children – a self-fulfilling prophecy. This may not have happened if the teacher did not hold the belief in the first place.

For the remainder of this discussion, I will refer to children as either having mathematical proficiency at or below grade-level expectations, or beyond grade-level expectations.

Recognising that children in a typical class represent a range of levels of mathematical proficiency, we claim that whole class teaching cannot be effective. We hold this position for all subjects, not just mathematics. Teaching a class of children who are at different

proficiency levels as if they are all at the same level leaves most of the learners in the class unable to participate in and benefit from the lesson.

The NumberSense Mathematics Programme was developed to support teachers using the programme to provide learning opportunities that are matched to the proficiency levels of the children in their class. The workbooks were designed to be implemented at the rate of a page-a-day. With each page consisting of coherent, clearly defined tasks for children to complete each lesson. In addition, the expectation is that if children are working in the workbook best suited to their level of proficiency, they will be able to work independently to complete the page set by the teacher for the day. Against this background, it is possible to imagine a class in which each child is working on the page in the overall learning journey that is best suited to their proficiency level, their age, their grade, and the number range in which they can work with confidence.

To have every child in the class on a different page of the journey is manifestly unfeasible. However, for teachers to group the children into three (or two) groups of children who are more similar in their level of proficiency and working with them at their level is both a reasonable expectation and is practical. Many teachers do this with great success every day.

The purpose of this discussion paper is to address some of the classroom realities that are emerging as teachers use the NumberSense Programme to provide differentiated learning opportunities. At the same time the purpose is also to address the implications of the recently introduced Comprehensive NumberSense Workbooks (2024) which address the complete (or full) school curriculum for mathematics.

The reality of differentiation in NumberSense Mathematics classrooms

There is abundant evidence that many of the schools and teachers implementing the NumberSense Programme are doing so in classes where children are assigned to groups according to their proficiency levels/needs. Although the confidence and reliability with which teachers are doing the allocation is uneven, most are making efforts in this regard.

Typically, teachers assign children to one of three groups according to their perception of the children's proficiency levels/needs. During a mathematics lesson they will assign

different workbook pages to each of the groups to complete. Working with one group at a time, the teacher then provides the children with the support that they need for the page that they are busy with. Generally, the workbook page range across the three groups in a class is within a workbook.

There are, however, many instances in which the group whose proficiency level is behind what we expect of them, could be as many as three or even four workbooks behind the age and grade appropriate workbook. While we want children to have learning opportunities and to be supported at their proficiency level, deliberate efforts also need to be made to close the gap between where children are and where they should be in terms of age-appropriate norms. The challenge for teachers is to work with these children in such a way as to close the gap between their proficiency level and the proficiency level we expect them to be at. There is, sadly, very little evidence that this is happening. In most cases the gap in the proficiency levels in a class is increasing.

This situation has been brought into sharp focus with the introduction of the Comprehensive NumberSense Workbooks. These workbooks address the full curriculum for the year and, each year children should complete all four of the workbooks for that year. Children who start the year three or more workbooks behind where they should be in terms of their proficiency level, are now neither closing the gap nor completing the full curriculum.

Rethinking differentiation in the NumberSense Mathematics classroom

Against this background I want to propose a rethinking of what it means to provide differentiated instruction in a NumberSense Mathematics classroom using the Comprehensive Workbooks. This rethinking is guided by both the need to finish the workbooks of the year during the year in order to complete the curriculum for the grade, and, the need to reduce the gap in proficiency levels in the class.

Children should be assigned to three groups (not necessarily equal in size) according to their proficiency levels. However, in contrast to previous recommendations, each of the groups will be working on the same page of the same workbook on the same day.

The group whose mathematical proficiency is beyond what we expect of them should not be assigned to the next workbook. These children do not need more of the same (moving further along the workbook journey). They need additional, different, activities that match

their developmental level and develop curiosity. These children need less of a teacher-led introduction to the page of the day before they can work independently on the page. That said, the expectation of these children is that they will engage in deep, rich reflection of the page with the teacher after they have completed the page. An expectation that they should be aware of.

In the case of the children whose mathematical proficiency is behind what we expect of them, the teacher will spend more time working with the group during the teacher-led activity before the group works independently to complete the page of the day. With this group, the teacher-led activity may involve the teacher doing selected tasks or activities on the page with the group. Whereafter, she may identify tasks or activities that the children must do during the independent work session and others that the children should at least attempt or may even leave out. With this group the reflection on the page will focus more on checking solutions and discussing common errors.

Rethinking differentiation in the NumberSense Mathematics Classroom in no way suggests a move away from the traditional NumberSense lesson structure (teacher-led activity, independent written work, and reflection), but rather a redistribution of time allocated to each component of the lesson for each group. Figure 1 conveys how differentiation in a NumberSense classroom with all the children on the same page of the same workbook on the same day manifests. Note, the figure illustrates the proportional allocation of time to each of the lesson activities and not the flow or sequencing of the lesson. That is the subject of a different discussion.

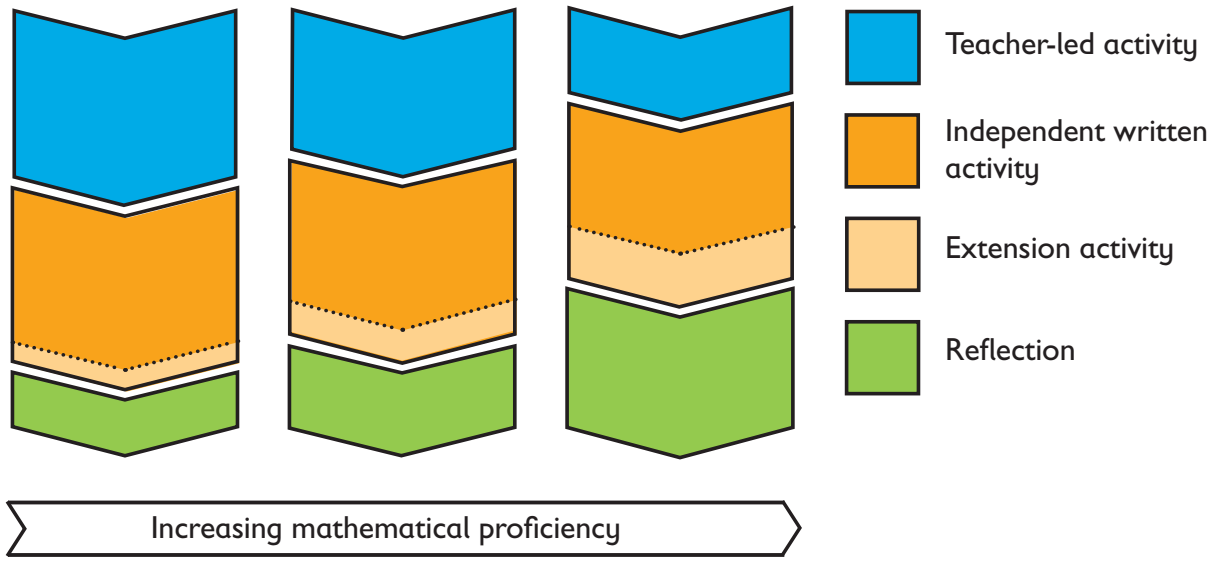


Figure 1: Allocation of time to the different components of a NumberSense Mathematics lesson in relation to the mathematical proficiency level of the groups

In summary, differentiation is not so much in terms of the content (the page of the day) but in the extent and nature of the support that the teacher provides to the children. *Figure 2* illustrates the changing nature of the teacher's role in the teacher-led and reflection activities for the different groups.

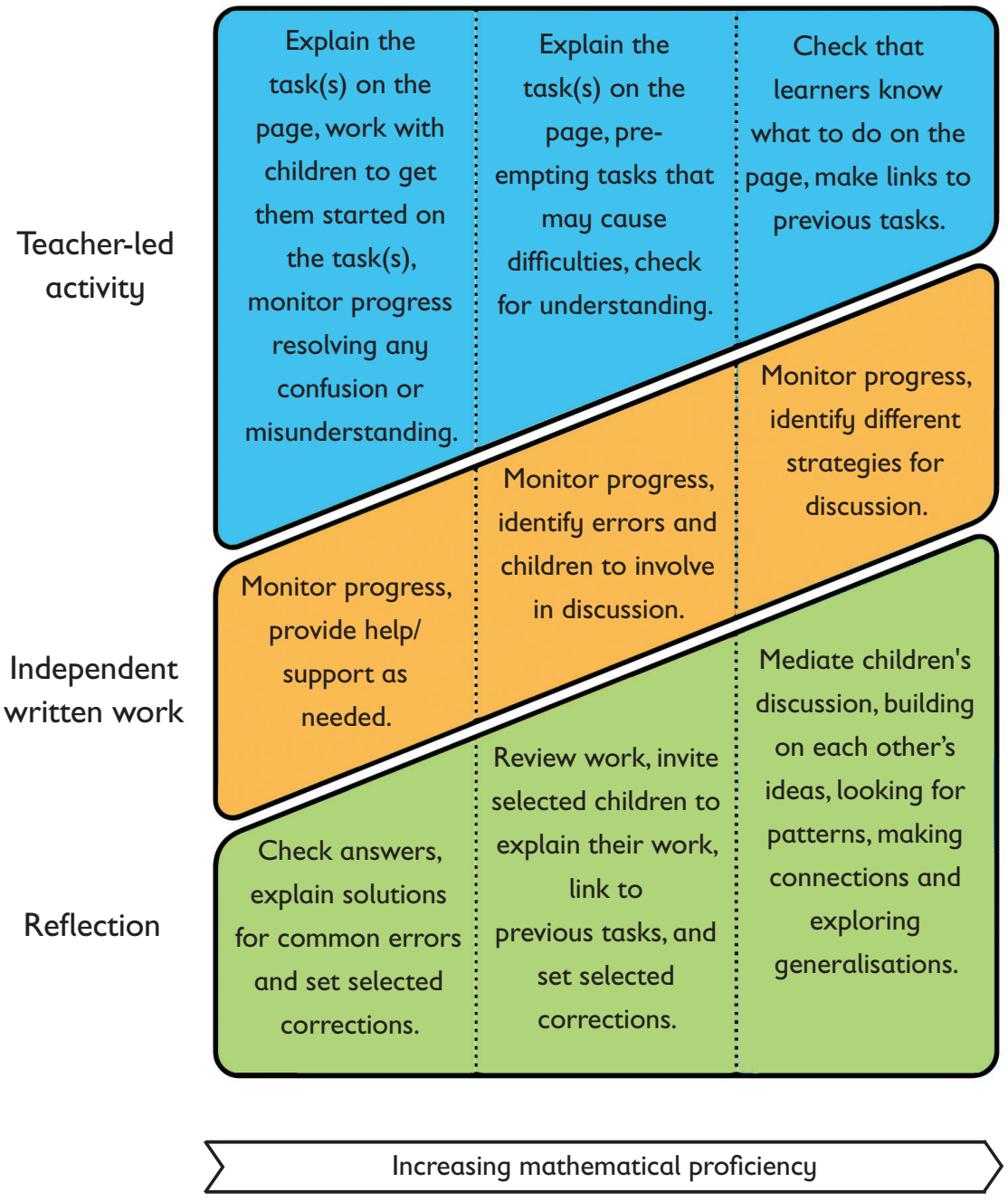


Figure 2: The changing role of the teacher in the teacher-led, independent written work and reflection activities of a NumberSense Mathematics lesson in relation to the mathematical proficiency level of the groups.

The need for extension activities

What *Figure 1* highlights is the expectation that children whose level of mathematical proficiency exceeds what we expect of them may well complete the assigned page before the time allocated to the independent written activity. These children need additional, different activities, activities that provide challenge, that encourage the use of a range of problem-solving strategies (*Figure 3*) and develop curiosity. For the purpose of this discussion we will refer to these as extension activities.

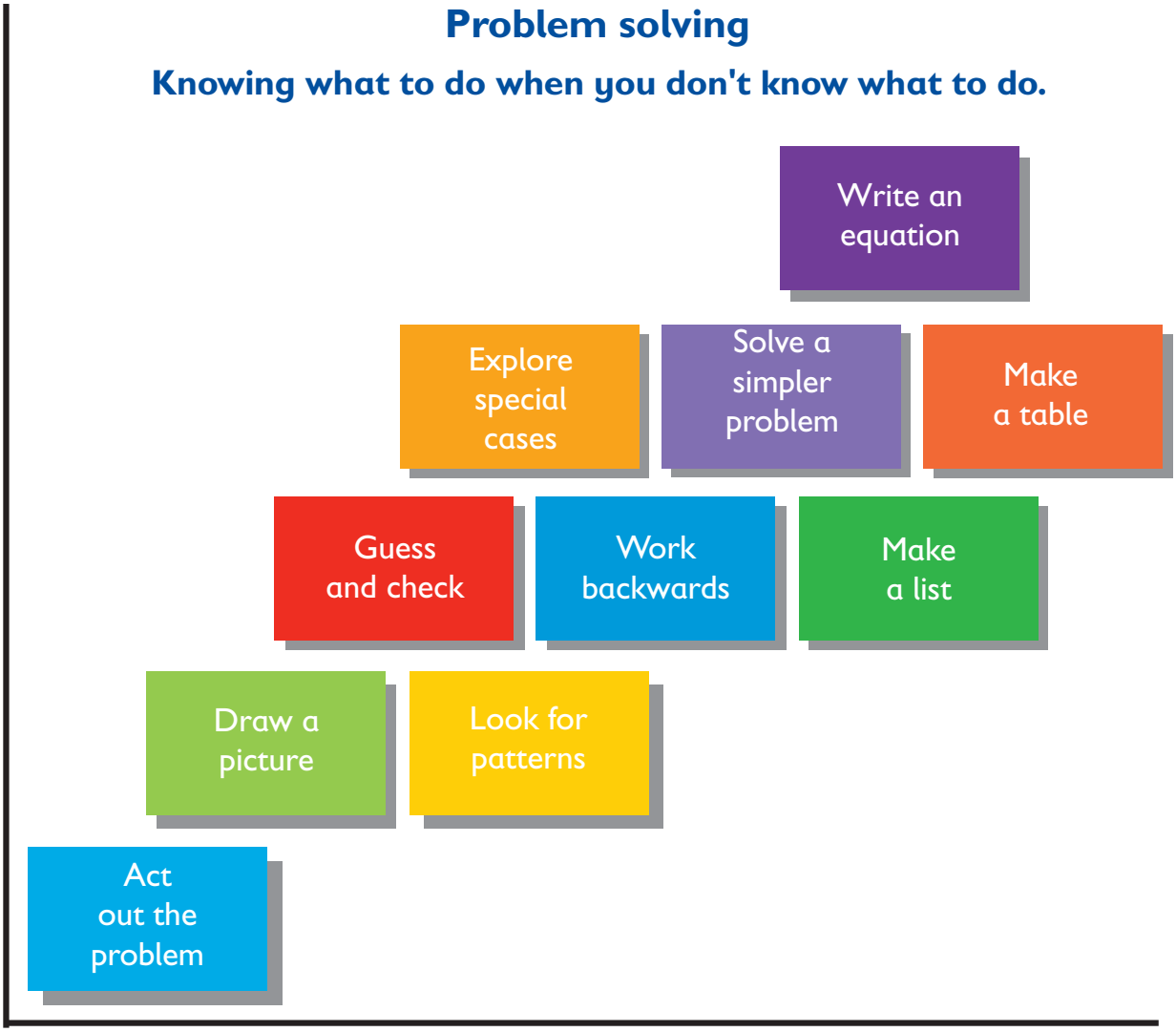


Figure 3: Problem solving strategies.

Extension activities/problems

Extension activities should be designed to enrich the learning experience and challenge children beyond the curriculum expectations of the grade. Beyond the curriculum expectations of the grade does not mean the expectations of the next grade, but rather activities that develop diverse problem-solving skills and encourage curiosity.

Extension activities/problems should:

- Be challenging, not the application of familiar procedures.
- Encourage creativity and problem-solving.
- Provide opportunities for children to investigate and identify patterns and mathematical structure.
- Be interesting and engaging, motivating children to want to solve the problem.

Developing extension activities/problems

Teachers are encouraged to develop a library of extension activities (problems). Two excellent resources that are freely available are the past papers of the:

- South African Mathematics Foundation (SAMF) Mathematics Competition, formerly known as the AMESA 4 to 7 challenge
  www.samf.ac.za/en/sa-maths-challenge-past-question-papers-solutions
- The University of the Witwatersrand (WITS) mathematics competition
  wmc.ms.wits.ac.za/past-papers/#page-content

Teachers can use these past papers to identify and select problems that are appropriate to the children in their class. It is recommended that these problems are reproduced on small pieces of paper that children can take and paste into a dedicated extension activity booklet.

Another source of challenge problems are the so called Ken Ken puzzles. These are very effective in developing basic calculation skills and logical reasoning.

- Ken Ken puzzle
  www.kenkenpuzzle.com/#

Problems and selected Ken Ken puzzles can be posted on a board in the class on a weekly basis (*Figure 4*).

Figure 4: Extension activities

As children complete the page in the NumberSense Workbook, they can take a problem or puzzle from the board, paste it in their extension activity notebook and complete the problem. Using an extension activity notebook provides a record of the problems that the child has attempted and the basis for discussion of the problem(s) between children and teachers.

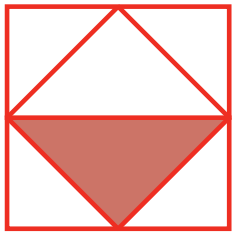
It is important that the weekly problems posted by the teacher are graded so that children in each of the proficiency level groups can attempt a problem when they have completed the NumberSense Workbook page of the day.

Aarnout Brombacher (2024)

A group of children is standing, evenly spaced, in a circle. Each child is numbered consecutively, starting at 1. Child number 5 is standing in the circle directly opposite child number 18. How many children are in the circle?

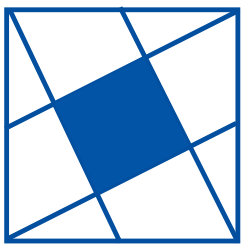
SAMC (2019), Third round, Grade 4 & 5

The figure shows a square, the midpoints of the sides of the square are joined.
What fraction of the figure is shaded?



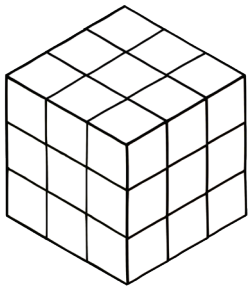
WITS Mathematics Competition (2022), Qualifying round, Middle Primary

The vertices of a square are connected to the midpoints of the opposite sides. The area of the square is 650 cm^3.
What is the area of the shaded part in square centimetres?



SAMC (2019), Third round, Grade 6 & 7

The six surfaces of this cube are painted red. If the cube is then cut into 27 equal-sized smaller cubes and taken apart, how many unpainted surfaces are there?

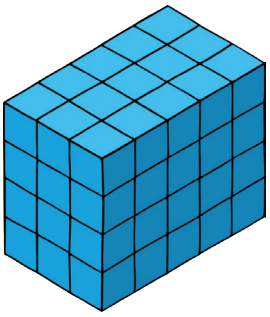


SAMC (2019), Third round, Grade 6 & 7

A 20 × 20 × 20 cube is constructed by sticking 8 000 unit cubes together. The cube is subsequently dropped in paint and then broken up into the original 8 000 cubes.
How many cubes have exactly 1 painted side?

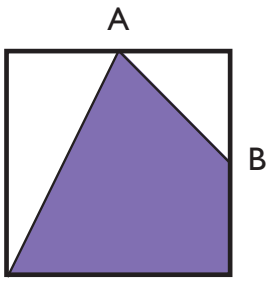
WITS Mathematics Competition (2019), Final round, Grade 6 & 7

This large cuboid is made up of 60 small cubes. If the whole cuboid is dipped into blue paint, how many of the small cubes would have only one face that is blue?



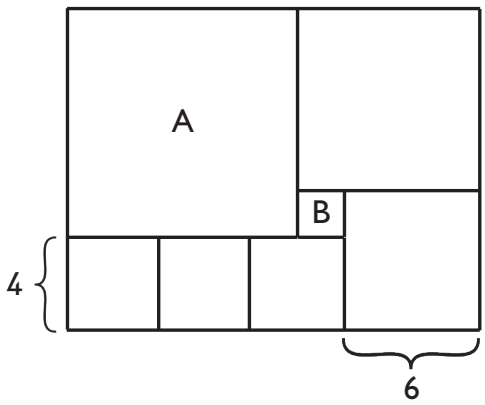
SAMC (2021), Third round, Grade 4 & 5

Points A and B are the midpoints of two different sides of this square. What fraction of the square is shaded?



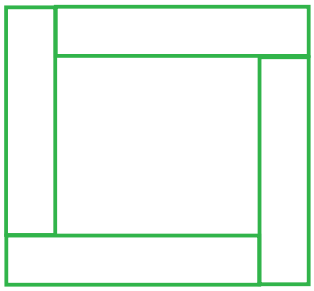
SAMC (2021), Second round, Grade 5

The figure shown consists of 7 squares with some lengths given. Square A is the biggest one and square B is the smallest one. How many squares of size equal to square B can square A be divided into?



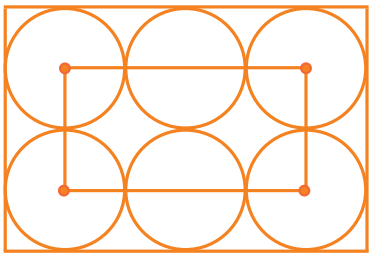
WITS Mathematics Competition (2019), Qualifying round, Grade 4 & 5

In the figure below, four identical rectangles form a square as shown. The perimeter of each rectangle is 40 cm. What is the area of the whole figure?



SAMC (2019), Second round, Grade 7

There are six identical circles in the figure. The circles touch each other as well as the sides of the larger rectangle. The vertices of the small rectangle are the centres of the circles. The perimeter of the small rectangle is 60 cm.
What is the perimeter of the larger rectangle?

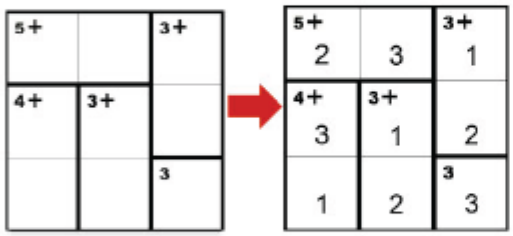


SAMC (2023), Second round, Grade 7

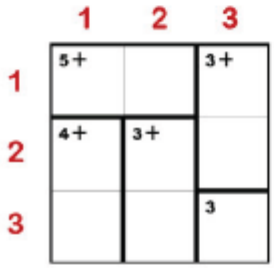
KENKEN CLASSROOM
Puzzles That Make Math Fun.

HOW TO SOLVE KENKEN®

1. Your goal is to fill in the whole grid with numbers, making sure no number is repeated in any row or column.



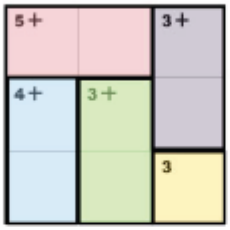
2. In a 3x3 puzzle, use the numbers 1 – 3.



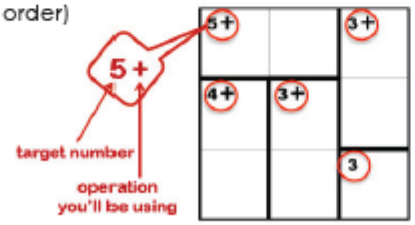
In a 4x4 puzzle, use the numbers 1 – 4. In a 5x5, use the numbers 1 – 5, and so on.

3. The heavily-outlined areas are called "cages."

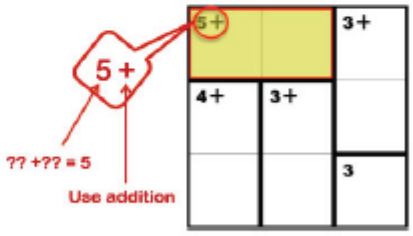
This puzzle has 5 cages.



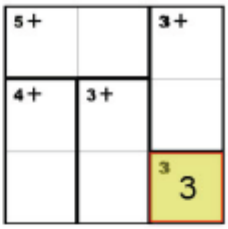
4. The top left corner of each cage has a "target number" and math operation. The numbers you enter into a cage must combine (in any order) to produce the target number using the math operation noted (+, -, ×, or ÷).



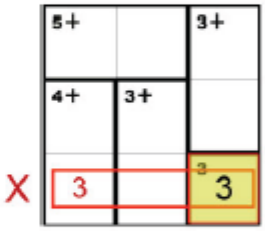
5. In this cage, the math operation to use is addition, and the numbers must add up to 5. Since this cage has 2 squares, the only possibilities are 2 and 3, in either order (2 + 3 or 3 + 2 = 5).



6. A cage with one square is a "Freebie"... just fill in the number you're given.



7. A number cannot be repeated within the same row or column.



HINT:

Each puzzle has one unique solution.

Extension activities

www.kenkenpuzzle.com/#

A.

2	4+	
4+		5+
3+		

B.

2−	5+	
	5+	2−

C.

2×		3×
6×		
	6×	

D.

9×	2÷	
		6×
2×		

E.

9+		5+	
	1	5+	7+
5+			
	7+		1

F.

16×		6×	
	2÷	12×	
3			2÷
6×			

G.

3−	7+		1−
	1−		
3+		11+	
5+		1	

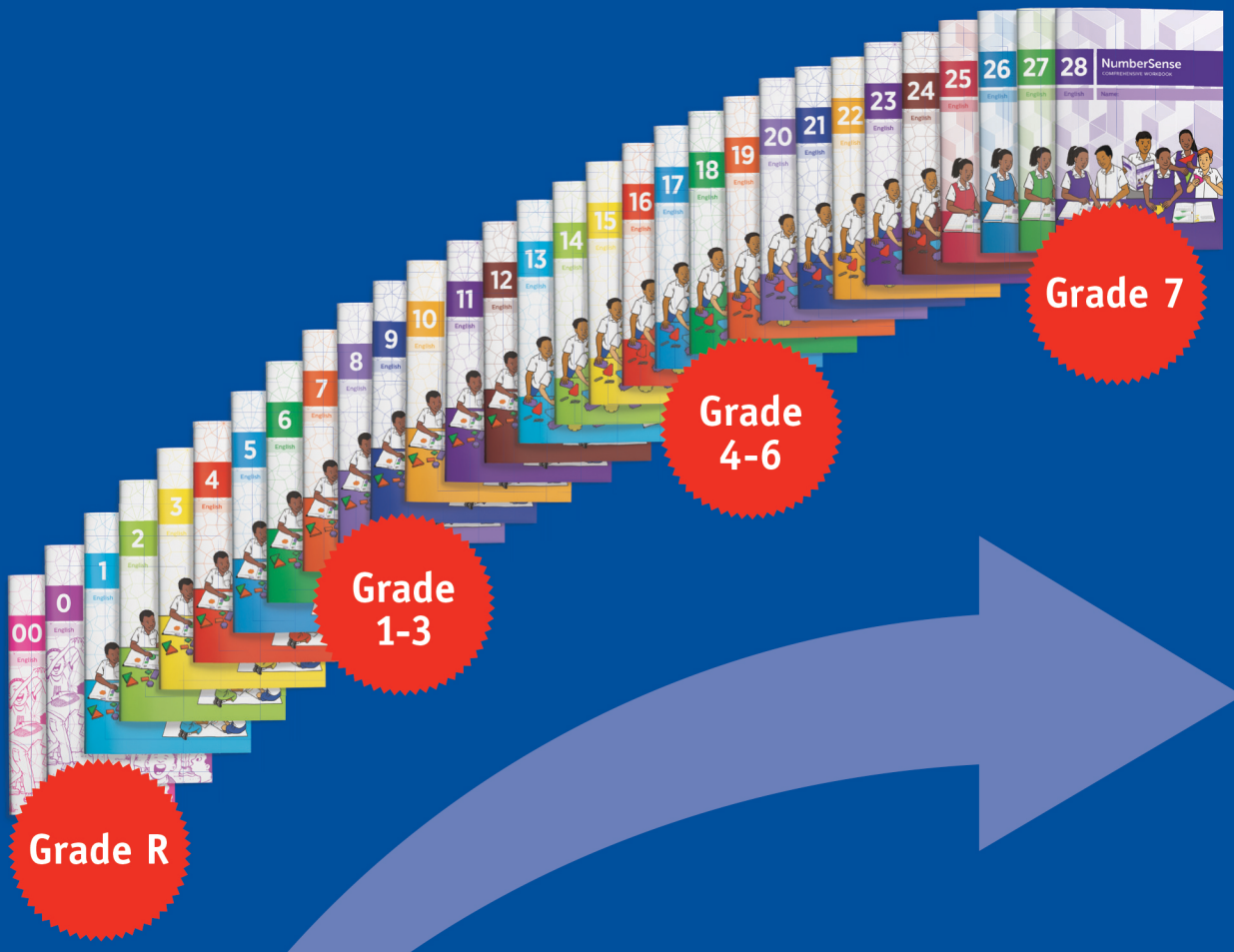
H.

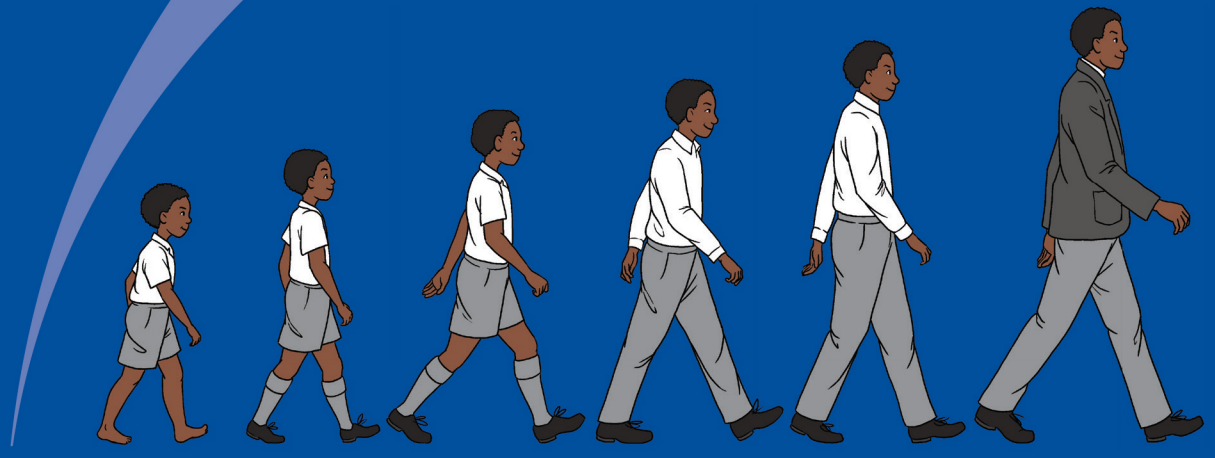
3×	2÷		9+
2÷	12×		2
		2−	

I.

6+	6+		4+	10+
	9+			
3	3+		9+	
9+				5+
5+		6+		

NUMBERSENSE COMPREHENSIVE WORKBOOK JOURNEY





Brombacher & Associates

A. Unit E23, Prime Park, Mocke Road, Diepriver
T. (021) 706 3777
E. info@NumberSense.co.za / info@Brombacher.co.za

www.NumberSense.co.za
www.Brombacher.co.za